NRL Report 7937

Some Comments on Power Combiner/Switch Matrices

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November 26, 1975

NAVAL RESEARCH LABORATORY
Washington, D.C.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 7937	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SOME COMMENTS ON POWER COMBINER/SWITCH MATRICES		5. TYPE OF REPORT & PERIOD COVERED An interim report on one phase of the NRL problem. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C. W. Young, Jr., and M. L. Reuss, Jr.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem R08-36 Project XF 54-545-007, Task K07
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Naval Electronic Systems Command Washington, D.C. 20360		12. REPORT DATE November 26, 1975 13. NUMBER OF PAGES 27
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Matrix combiner
Switch isolation
Hybrid unbalance
Phase errors
Power variations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

High-power signals can be switched with low-power switch control using hybrid matrices and parallel amplifiers. This study treats the effects of input phase and amplitude errors on the outputs of combiner/switch matrices. Worst-case isolation and loss are considered as a function of the errors mentioned and of the unbalance of the hybrids forming the matrix. Effects of each parameter are considered individually; a computer program to study the effects of several errors simultaneously is listed in the report.

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CONTENTS

SOME COMMENTS ON POWER COMBINER/SWITCH MATRICES

INTRODUCTION

The feasibility of using hybrid matrices and parallel amplifiers to increase output power over broad bandwidths has been established. It has also been established that high-power signals can be directed to different output ports by switching at low power levels prior to the amplifiers [1,2]. Low-power switch control gives the potential [1] of fast (diode) switching of the high-power output. Published work on power combining has dealt primarily with loss in a matrix power combiner due to imperfect components. However, when the switch aspect is added to a power combiner, another parameter — isolation between output ports — becomes important for many applications. This study will primarily be concerned with the effect of component imperfections or errors on overall switch isolation.

CIRCUIT DESCRIPTION

Typical power combiner/switch circuits are shown in Fig. 1 and consist of an input-power-divider circuit, switch control elements, parallel amplifiers, and a power-combiner matrix. Figure 1a depicts a power combiner/switch whose output port is determined by selection of the appropriate input port, and Fig. 1b depicts a power-combiner switch controlled by phase shifters located prior to the amplifiers. If phase shifters are used as the control elements, there are many possible variations of the input power divider.

Due to the large number of potential combinations of power-dividers and control elements and the high probability that the parallel amplifiers will be operating at saturation, only the effect of errors directly relating to the inputs of the combiner matrix and the matrix proper will be considered. In the main body of this report these errors will be considered individually, (input phase errors with no other errors, hybrid unbalance with no other errors, etc.), and in Appendix A a listing of a computer program to investigate different simultaneous errors is included.

ASSUMPTIONS

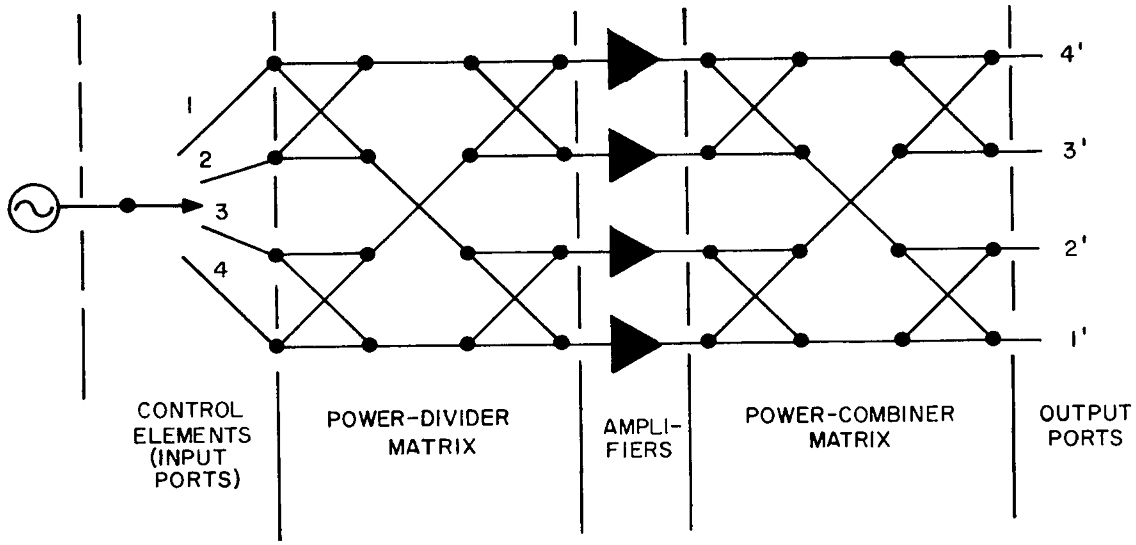
Although 3-dB hybrids having quadrature phase outputs or hybrids having a 180-degree phase relation (such as magic tees) may be employed to form the output matrix, it will be assumed for this study that the hybrids employed are ideal quadrature hybrids with the exception that the coupled signal need not be equal to the transmitted signal. Each hybrid will be assumed to have infinite isolation, precisely a 90° phase difference

Manuscript submitted October 2, 1975.

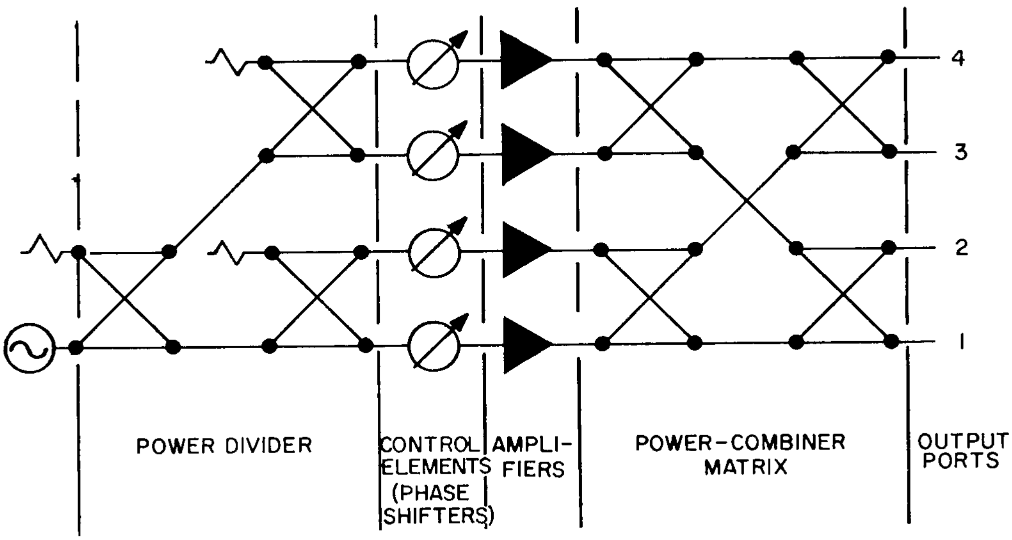
between outputs, no loss, and a perfect match. Thus the voltage coupling coefficient (C) and the voltage transmission coefficient (T) will, via conservation of energy, have the relationship

$$T^2 + C^2 = 1,$$

where T and C are real.



(a) Control by the input port selected



(b) Control by phase shifters

Fig. 1 — Typical power combiner/switch circuits

All hybrids in the matrix will be assumed identical and, unless otherwise specified, line lengths connecting the hybrids will be considered equal. For the ideal case, there will be assumed to be at each input port of the matrix equal-amplitude signals and appropriate phase values to direct power to the desired output port. Combiner/switch matrices having N input ports and N output ports will be considered, with N being limited to

$$N = 2^k,$$

where k is an integer.

Deviation of the input signals from the ideal case or imperfection of the components in the matrix can result in a "scattering" of the input energy to other than the principal output port. This scattering can reduce the power at the principal port relative to ideal situation. This reduction of power at the principal port will be termed loss.

The isolation of the output matrix will be defined for purposes of this study as the ratio of power at the principal output port to the power at the port having the next highest output power. The isolation will be expressed in decibels.

INPUT ERRORS

Phase and amplitude errors present at the output of the amplifiers can be a major source of loss and degraded switch isolation.

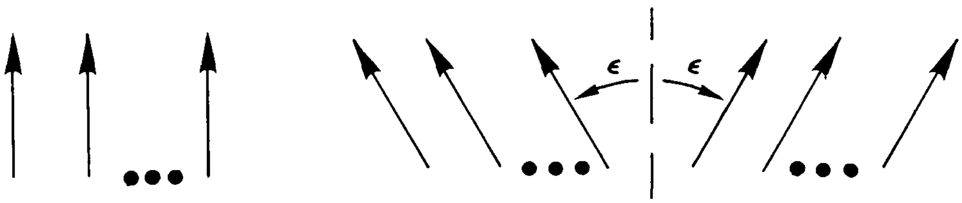
Phase Errors

Deviation from the phase values required at each input port to direct the energy to a given output port is a major factor in degrading the performance of an otherwise ideal power combiner/switch. The worst-case loss (Fig. 2b) with phase errors limited to $\pm\,\epsilon$ would occur when $N/2$ of the input voltages deviate from the required values of phase by a positive amount ϵ and the remaining $N/2$ input voltages deviate from the required values of phase by a negative amount $-\epsilon$. The ideal case losswise (Fig. 2a) is that of $\epsilon = 0$. The resultant of the vectors in Fig. 2b would be less than in the ideal case of Fig. 2a. The insertion loss at the principal output port caused by this type of phase error is

$$\text{worst-case loss (dB)} = -10 \log_{10} \left\{ \frac{N/2(\cos \epsilon) + N/2[\cos(-\epsilon)]}{N} \right\}^2$$

$$= -10 \log_{10} (\cos \epsilon)^2.$$

This worst-case loss (which does not include ohmic losses) is independent of the size of the matrix.



(a) Ideal case (b) Worst case for phase errors limited to $\pm\epsilon$

Fig. 2 — Signals at the desired output port from each of the input ports

The effects of worst-case phase error on isolation can best be seen by considering a single output port other than the principal output port when half ($N/2$) of the signals have an effective phase error of ϵ and the other half $-\epsilon$. Ideally ($\epsilon = 0$) $N/2$ of the signals would be out of phase with the remainder (Fig. 3a), thus yielding a resultant of zero, since the signal components from each of the inputs are of equal amplitude. In the worst-case condition ($\epsilon \neq 0$) superposition of the signals with the phase condition shown in Fig. 3b yields the largest resultant. The power out of this secondary port relative to the total input power is

$$\text{power out (dB)} = 10 \log_{10} \left[\frac{N/2\ (\sin \epsilon) + N/2 \sin (180 - \epsilon)}{N} \right]^2$$

$$= 10 \log_{10} (\sin \epsilon)^2 .$$

Thus the worst-case isolation for input phase errors up to $\pm\epsilon$ becomes

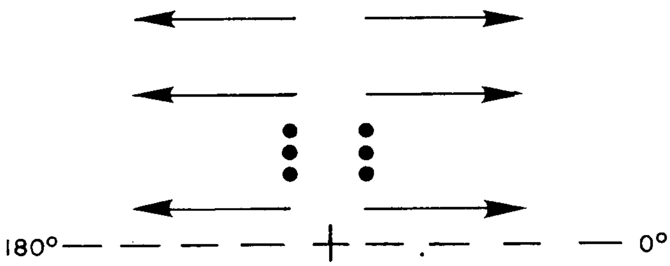
$$\text{worst-case isolation (dB)} = 10 \log_{10} (\cos \epsilon)^2 - 10 \log_{10} (\sin \epsilon)^2$$

$$= -10 \log_{10} (\tan \epsilon)^2 .$$

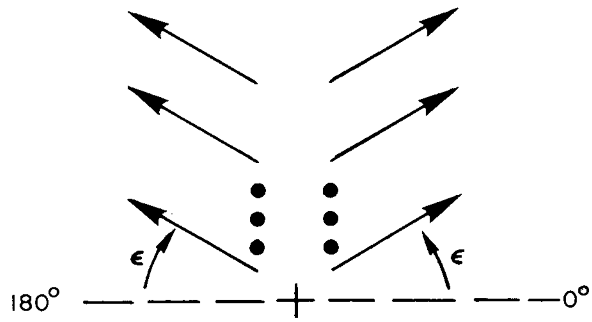
For both insertion loss and isolation the worst-case input phase error contribution is independent of the size of the matrix: curves showing the relationship between loss, isolation, and phase error are plotted in Fig. 4. It is evident that as the size of the output combiner matrix is increased, the probability of the worst-case condition occurring would decrease.

Amplitude Variation

Input amplitude variations of two types were considered. In the first type half of the input power levels were X dB above a uniform input level and the other half of the inputs were X dB below the same uniform level. In the second type one input power level was lower than the $N - 1$ remaining inputs, which were of uniform level.



(a) Ideal case



(b) Worst case for phase errors limited to $\pm\epsilon$

Fig. 3 — Signals at an output port other than the desired output port from each of the input ports

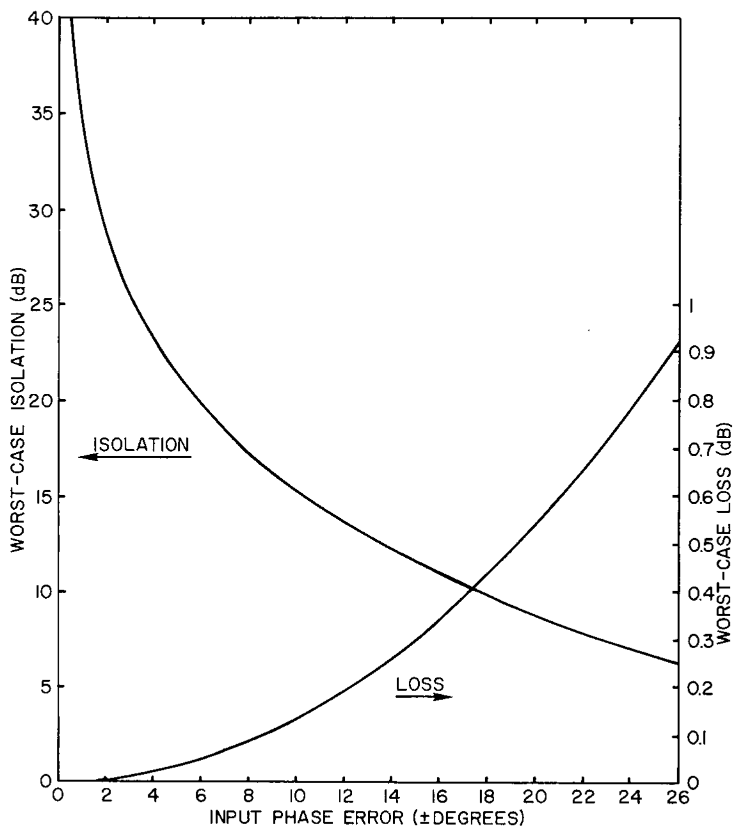


Fig. 4 — Relationship of worst-case isolation and worst-case loss as a function of the input phase error

When the amplitudes of the inputs are such that half of the power levels are X dB above a nominal value and half are X dB below the same nominal value, the total input power will be greater than with a uniform input at the nominal value. If the variation X of input power is ±0.5 dB, ±1 dB or ±2 dB, the total input power will be 0.03 dB, 0.11 dB, or 0.45 dB respectively greater than with a uniform distribution. Vector addition of the signals at the principal output indicates that the power at this port relative to the power at this port with a uniform input distribution (with each input power being normalized to unity and the total input thus being N), will be

$$\text{power out (dB)} = 10 \log_{10} \left[\frac{\left(\frac{1}{\sqrt{2}}\right)^k (N/2) \left(Y + \frac{1}{Y}\right)}{N} \right]^2$$

$$= 20 \log_{10} \frac{Y + \frac{1}{Y}}{2} ,$$

where the voltage factor $Y = 10^{X/20}, (1/Y) = 10^{-X/20}$, and $(1/\sqrt{2})^k = 1/\sqrt{N}$, in which $1/\sqrt{2}$ is the voltage transfer factor for each of the k hybrids in series. This expression results in output powers at the principal port 0.01 dB, 0.06 dB, and 0.23 dB greater than with uniform input when power variations are ±0.5 dB, ±1 dB, and ±2 dB respectively. Comparing these values with the corresponding increase in input power relative to a uniform input distribution and considering conservation of energy, the only place for this difference in power to exit is at the other (isolated) output ports of the matrix. Thus the principal effect of the variation in the input power is to decrease the isolation between the principal output port and the other output ports.

The worst-case isolation will result at the output port where all the signal components having $+X$-dB input are out of phase with the signal components having $-X$-dB input. Thus worst-case isolation is of the form

$$\text{worst-case isolation(dB)} = 10 \log_{10} \left[\frac{(N/2)\left(Y + \frac{1}{Y}\right)}{N} \Big/ \frac{(N/2)\left(Y - \frac{1}{Y}\right)}{N} \right]^2$$

$$= 20 \log_{10} \left[\left(Y + \frac{1}{Y}\right) \Big/ \left(Y - \frac{1}{Y}\right) \right].$$

Figure 5 indicates the dependence of this worst-case isolation on worst-case amplitude variation (half of the inputs having a $+X$-dB power difference from the nominal input and the other half having a $-X$-dB power difference from the nominal).

The second type of amplitude variation in an ideal matrix with ideal input phase values is that of one input to the combiner matrix being smaller than the remaining $N - 1$ inputs, which are of equal amplitude. This case should be indicative of a single amplifier failing powerwise. Since specification of a power combiner/switch would be under normal operating conditions, the degradation of output power will be treated as the ratio of power out of the principal port to the total input power when all N input powers are equal.

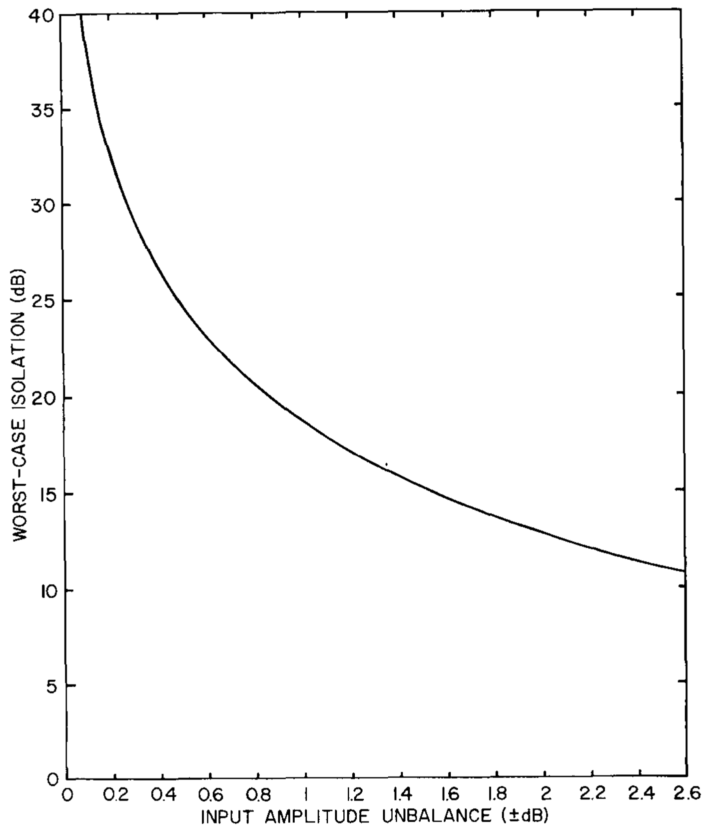


Fig. 5 — Worst-case isolation when half of the input amplitudes are above a nominal value and the other half are below the nominal value

In this case, as the power at the nonuniform or "defective" input port decreases by X dB, the voltage W at this port is defined by $W = 10^{-X/20}$, and the total input power for the ideal case would be N, since the input powers are normalized to unity. The degradation of power output as previously defined would be

$$\text{power degradation (dB)} = -10 \log_{10} \left\{ \frac{\left(\frac{1}{\sqrt{2}}\right)^k [(N - 1) + W]}{N} \right\}^2$$

$$= -20 \log_{10} \frac{(N - 1) + W}{N}$$

at the principal port. Isolation at the secondary ports would be

$$\text{isolation (dB)} = 10 \log_{10} \left\{ \frac{\left(\frac{1}{\sqrt{2}}\right)^k \left[(N - 1) + W\right]}{\left(\frac{1}{\sqrt{2}}\right)^k \left[\frac{N}{2} - \left(\frac{N}{2} - 1\right) - W\right]} \right\}^2$$

$$= 10 \log_{10} \left[\frac{(N - 1) + W}{1 - W}\right]^2$$

$$= 20 \log_{10} \left(\frac{N}{1 - W} - 1\right) .$$

The isolation defined here is between the principal port and any of the other output ports.

Figure 6 depicts the effect of power reduction at a single input on both power degradation and isolation. Both of these quantities are a function of the size of the combiner/switch matrix.

MATRIX ERRORS

Hybrid Unbalance

In an ideal matrix there is no loss from the 2^k inputs to the single output. However, if there is an unbalance in the hybrids ($T \neq C$, where T and C are the voltage transmission and coupling coefficients), there is a "scattering" of power to the multiple outputs that will reduce the power at the principal output port and decrease the isolation between the principal and secondary outputs. Hybrid unbalance will be defined as

$$\text{unbalance (dB)} = 10 \log_{10} \frac{C^2}{T^2} .$$

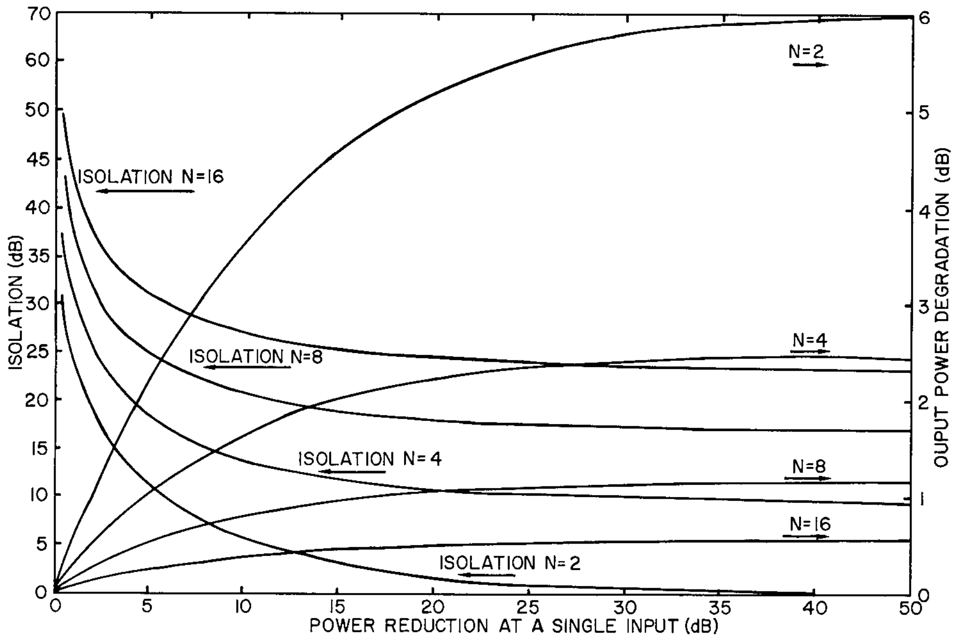


Fig. 6 — Effect of power reduction at one of N input ports on isolation and power degradation

A heuristic approach to determining the effects of hybrid unbalance on worst-case loss and isolation is presented in Appendix B. This approach leads to the expressions

$$\text{worst-case loss (dB)} = -20k \log_{10} \frac{C + T}{\sqrt{2}}$$

and

$$\text{worst-case isolation (dB)} = 10 \log_{10} \left(\frac{C + T}{C - T}\right)^2,$$

where the number of output ports is $N = 2^k$.

The expression for loss due to hybrid unbalance indicates that as the number of output ports increases, the unbalance of the hybrids becomes a more significant factor in determining the loss. However, the worst-case isolation is independent of the number of output ports. Figure 7 illustrates worst-case isolation and typical worst-case loss curves for power combiner/switch matrices as a function of hybrid unbalance.

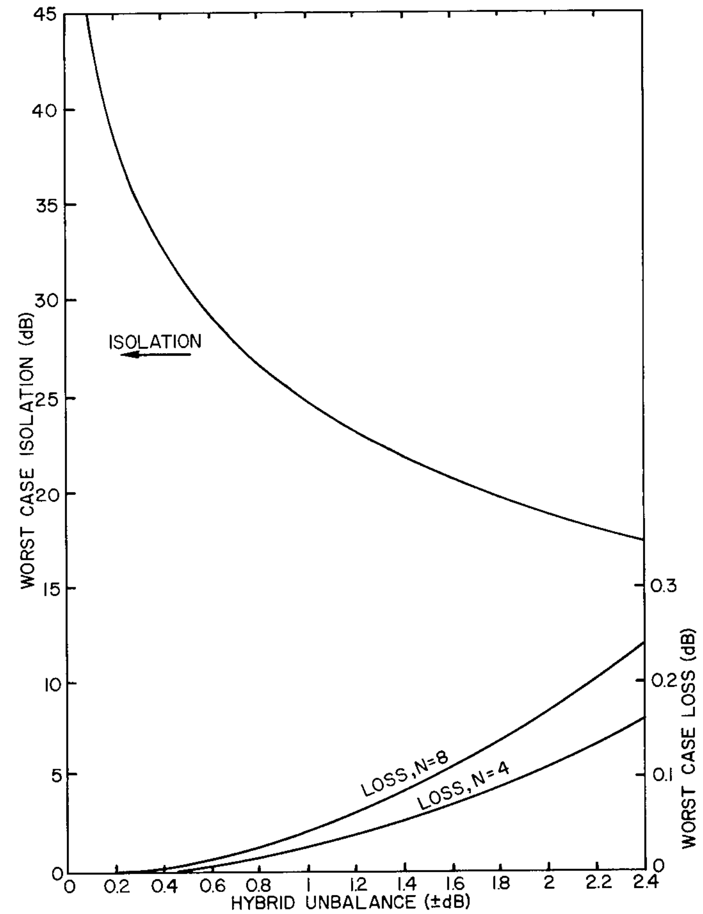


Fig. 7 — Effect of hybrid unbalance on the worst-case isolation and, for typical numbers of output ports, on the worst-case loss

When only two of N outputs (where N is four or greater) are used, the two output ports may be selected to minimize the effect of hybrid unbalance on isolation. The best isolation that can be obtained via port selection with unbalanced output hybrids appears to be

$$\text{best-case isolation (dB)} = 10k \log_{10}\left(\frac{C+T}{C-T}\right)^2.$$

However, unless the hybrid unbalance is exceedingly poor, other factors will probably dominate in limiting the isolation.

Output-Hybrid Limitations

Analysis of the loss and isolation effects of phase errors in the lines connecting the identical hybrids in the matrix is beyond the intent of this report. However inspection of phase errors between signals incident on the final-output hybrid can provide insight into the effects of these phase errors on isolation and the limitations imposed on a power combiner/switch matrix by the output hybrid.

When two signals of unit magnitude and different phases ($e^{j\alpha}$ and $e^{j\beta}$) enter an unbalanced hybrid ($T \neq C$), the respective output signals are

$$V_1 = Te^{j\alpha} + jCe^{j\beta}$$

and

$$V_2 = jCe^{j\alpha} + Te^{j\beta} .$$

If $\alpha = \beta + \pi/2 + \Delta$, where Δ represents a phase error from the desired (nominal) value, the signal at the principal port becomes

$$V_p = je^{j\beta}(Te^{j\Delta} + C)$$

$$= je^{j\beta}(T \cos \Delta + C + jT \sin \Delta),$$

with the corresponding power being

$$P_p = |je^{j\beta}|^2 \, [|T \cos \Delta + C|^2 + |jT \sin \Delta|^2]$$

$$= 1 + 2TC \cos \Delta .$$

The signal and power at the second (isolated) port under the same conditions are

$$V_s = e^{j\beta}(-Ce^{j\Delta} + T)$$

$$= e^{j\beta}(T - C \cos \Delta - jC \sin \Delta)$$

and

$$P_s = |e^{j\beta}|^2 \, [|T - C \cos \Delta|^2 + |-jC \sin \Delta|^2]$$

$$= 1 - 2TC \cos \Delta .$$

Isolation of the second port relative to the principal port then is

$$\text{isolation (dB)} = 10 \log_{10} \frac{1 + 2TC \cos \Delta}{1 - 2TC \cos \Delta} \ .$$

Figure 8 illustrates the isolation characteristics with respect to both hybrid unbalance and phase variations from the nominal. The phase error entering the output hybrid could consist of various line-length errors (for specific paths) and a contribution to phase error due to the matrix input signals. By reference to Fig. 4 (phase error as defined for Fig. 4 is one half that defined for Fig. 8) the effect of phase error alone can be determined. In essence the performance of this output hybrid establishes an upper limit for the worst-case performance from a power combiner/switch matrix.

A scheme which could increase the power-handling capability of a broadband power combiner/switch matrix is described in Appendix C.

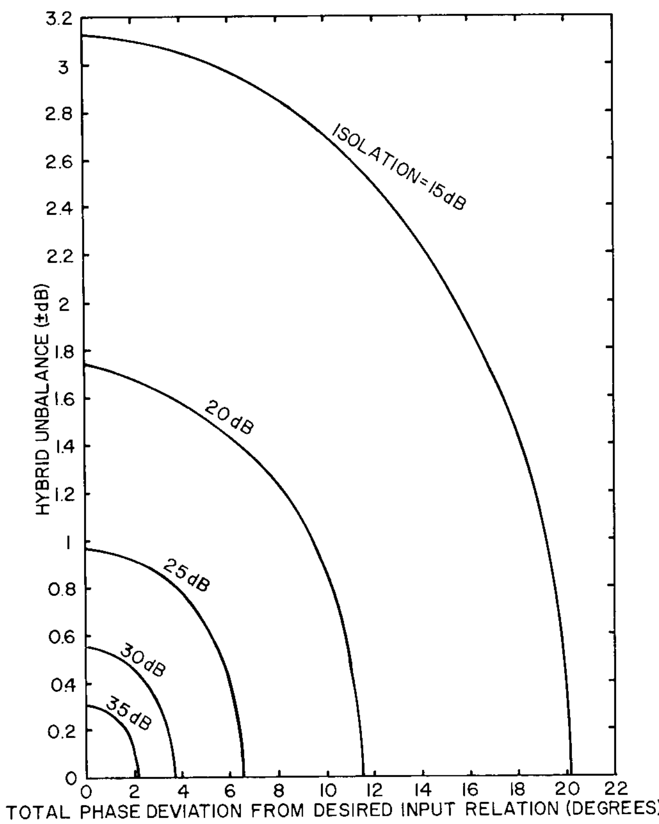


Fig. 8 — Joint effect of hybrid unbalance and phase deviation from the nominal on worst-case isolation

SUMMARY

The effects of certain types of error and certain component imperfections on the performance of idealized power combiner/switch matrices were considered individually. This combiner matrix consisted of idealized, identical hybrids, and each source of error was considered alone. Worst-case conditions were investigated for input phase errors, input amplitude errors, and hybrid unbalance. Isolation between output ports was found to be quite dependent on these factors.

In essence the errors studied individually indicate an upper limit for the worst-case performance that can be expected from a practical combiner matrix. Curves generated would be of use in establishing amplifier specifications for operation with these matrices.

REFERENCES

1. J.M. Miles and G.C. Page, "The Parallel Operation of Microwave Amplifiers Using Hybrid Couplers," NRL Report 7022, Mar. 3, 1970.

2. J.W. Sullivan, "Want More Power From Your TWT's ... Parallel 'em!," Microwaves **13** (No. 7), 38 (July 1974).

Appendix A
COMPUTER PROGRAM TO CALCULATE THE SWITCH LOSS AND ISOLATION FOR A COMBINATION OF ERROR TYPES

In the text combiner loss and switch isolation were derived for phase errors of the input signals, for amplitude errors of the input signals, and for unbalance of the hybrids. This was done in a rather straightforward manner by assuming the maximum magnitude for a single type of error and distributing these errors in such a manner as to obtain the worst-case performance: greatest loss and least isolation.

For more than a single type of error, say input amplitude variation and hybrid unbalance, the combiner/switch performance becomes considerably more difficult to analyze, since the output power at each of the N output ports is a function of which port is selected as the principal output (by proper phasing of the inputs) as well as the relative distribution of the various errors. A computer program was written to calculate the output power at each output port for any error distribution in both input power levels and input phasing plus any hybrid unbalance. The errors in power levels and phasing need not be distributed with equal magnitudes; provisions are also made to account for varying electrical line lengths between hybrids in the combiner matrix.

To use iterative calculations, the computer program assumes hybrid interconnections and port numbering as shown in the eight-port combiner of Fig. A1. The program is currently limited to a combiner matrix with $N = 8$ ports but can easily be extended to a matrix with any $N = 2^k$. The program does not calculate worst-case performance for a given set of parameter error limits; it calculates the power at each of the N outputs for a specific set of (program) input parameters. The input power to each of the N input ports is inputted as decibels, since only the *relative* magnitudes affect the performance. Likewise only the relative phases of the input signals are important, and similarly only the differences in the interconnecting line lengths between hybrids are important.

The computer program listing follows, together with an example of a printout resulting from arbitrarily chosen input parameters.

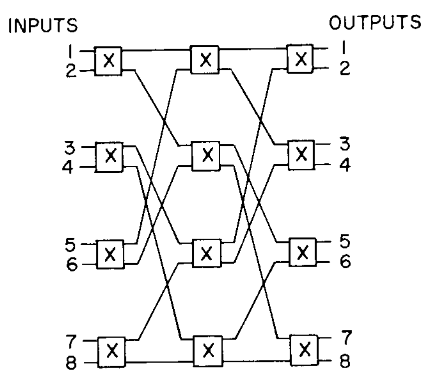


Fig. A1 — Hybrid interconnection for computer program

```
      PROGRAM COMBINE
      DIMENSION E(3,8),V(3,8),PWROUT(8),PWRINDB(8),PWRIN(8),THETA(8)
      DIMENSION ALPHA(2,8),PDBRAP(8),PDBRUI(8),PISOL(8)
      TYPE COMPLEX E,V,CPLEX
      PI=3.1415927
      RHO=360.0/(2.0*PI)
  101 READ 301,LOG2N
C LOG2N IS LOG BASE 2 OF THE NUMBER OF INPUTS
  301 FORMAT(I10,2F10.3,I10)
      IF(LOG2N.EQ.696969)STOP
      L2NM1=LOG2N-1
      N=2**LOG2N
      NHALF=N/2
C PROGRAM ASSUMES IDENTICAL PHASE QUADRATURE HYBRIDS
      READ 304,UNBDBST,DELUNB,NUNBAL
  304 FORMAT(2F10.3,I10)
      READ 302,(PWRINDB(I),I=1,8,1)
      READ 302,(THETA(I),I=1,8,1)
  302 FORMAT(8F10.3)
      DO 102 I=1,L2NM1,1
      READ 303,(ALPHA(I,J),J=1,N,1)
  303 FORMAT(8F10.2)
  102 CONTINUE
      IF(DELUNB.LT.1.0E-33)NUNBAL=1
      DO 199 IUB=1,NUNBAL,1
      UNBALDB=UNBDBST+(IUB-1)*DELUNB
      CY=POWRF(10.0,(UNBALDB/10.0))
      CSQR=CY/(1.0+CY)
      CSQRDB=-10.0*ALOG10(CSQR)
      T=SQRT(1.0-CSQR)
      C=SQRT(CSQR)
      CPLEX=C*(0.0,1.0)
      TOTPWR=0.0
      DO 111 J=1,N,1
      PWRIN(J)=POWRF(10.0,(-PWRINDB(J)/10.0))
      TOTPWR=TOTPWR+PWRIN(J)
  111 E(1,J)=SQRT(PWRIN(J))*CEXP(THETA(J)/RHO*(0.0,1.0))
      DO 114 I=1,LOG2N,1
      DO 112 J=1,NHALF,1
      V(I,2*J-1)=T*E(I,2*J-1)+CPLEX*E(I,2*J)
  112 V(I,2*J)=CPLEX*E(I,2*J-1)+T*E(I,2*J)
      DO 113 J=1,NHALF,1
      IF(I.EQ.LOG2N)GO TO 114
      E(I+1,2*J-1)=V(I,J)*CEXP(ALPHA(I,J)/RHO*(0.0,1.0))
  113 E(I+1,2*J)=V(I,J+NHALF)*CEXP(ALPHA(I,J+NHALF)/RHO*(0.0,1.0))
  114 CONTINUE
      POUTMAX=0.0
      DO 121 I=1,N,1
      PWROUT(I)=(CABS(V(LOG2N,I)))**2.0+1.0E-200
      PDBRAP(I)=10.0*ALOG10(PWROUT(I)/TOTPWR)
      PDBRUI(I)=10.0*ALOG10(PWROUT(I)/N)
      IF(PWROUT(I).GT.POUTMAX)POUTMAX=PWROUT(I)
  121 CONTINUE
      DO 122 I=1,N,1
  122 PISOL(I)=-10.0*ALOG10(PWROUT(I)/POUTMAX)
      PRINT 401
  401 FORMAT(1H1 *PROGRAM COMBINE*6X*CWY*/* THIS PROGRAM COMPUTES THE EF
     1FECT OF HYBRID UNBALANCE, INPUT AMPLITUDE AND PHASE ERRORS, AND*/*
     2 CONNECTING LINE LENGTH DIFFERENCES ON POWER COMBINER LOSS AND ISO
```

```
   3LATION*/)
    PRINT 402,N,UNBALDB,CSQRDB
402 FORMAT(* NUMBER OF INPUTS = *I2/* UNBALANCE = *F6.3* DB*8X*COUPLIN
   1G = *F6.3* DB*)
    PRINT 641,(J,J=1,N,1)
641 FORMAT(/10X*INTERCONNECTION PHASE ERRORS IN DEGREES*/14X*LINE NUMB
   1ER*8I10)
    DO 631 I=1,L2NM1,1
    PRINT 642,I,(ALPHA(I,J),J=1,N,1)
642 FORMAT(12X*SET NUMBER *I2,8F10.2)
631 CONTINUE
    PRINT 405,(I,I=1,N,1)
405 FORMAT(//14X *PORT NUMBER*8I10)
    PRINT 406,(PWRINDB(J),J=1,N,1)
406 FORMAT(/8X*INPUT POWER IN DB*8F10.3)
     PRINT 407,(PWRIN(J),J=1,N,1)
407 FORMAT(14X*INPUT POWER*8F10.3)
    PRINT 408,(THETA(J),J=1,N,1)
408 FORMAT(3X*INPUT PHASE IN DEGREES*8F10.2)
    PRINT 409,(PWROUT(J),J=1,N,1)
409 FORMAT(///13X*OUTPUT POWER*8F10.3)
    PRINT 410,(PDBRAP(J),J=1,N,1)
410 FORMAT(/4X*OUTPUT IN DB RELATIVE*8F10.3)
    PRINT 413
413 FORMAT(2X*TO ACTUAL TOTAL INPUT*)
    PRINT 411,(PDBRUI(J),J=1,N,1)
411 FORMAT(/4X*OUTPUT IN DB RELATIVE*8F10.3)
    PRINT 414
414 FORMAT(4X*TO TOTAL POWER WITH*/5X*EQUAL INPUT POWERS*)
    PRINT 412,(PISOL(J),J=1,N,1)
412 FORMAT(/* ISOLATION IN DB RELATIVE*8F10.3)
    PRINT 415
415 FORMAT(* TO PRIMARY OUTPUT PORT*)
199 CONTINUE
    GO TO 101
    END
       SCOPE
```

```
PROGRAM COMBINE      CWY
THIS PROGRAM COMPUTES THE EFFECT OF HYBRID UNBALANCE, INPUT AMPLITUDE AND PHASE ERRORS, AND
CONNECTING LINE LENGTH DIFFERENCES ON POWER COMBINER LOSS AND ISOLATION

NUMBER OF INPUTS =  8
UNBALANCE =  0,800 DB          COUPLING =  2,629 DB
```

INTERCONNECTION PHASE ERRORS IN DEGREES

LINE NUMBER	1	2	3	4	5	6	7	8
SET NUMBER 1	0.00	0.00	13.00	0.00	0.00	-6.00	0.00	0.00
SET NUMBER 2	0.00	-8.00	0.00	4.00	0.00	2.00	-3.00	5.00

PORT NUMBER	1	2	3	4	5	6	7	8
INPUT POWER IN DB	0,000	0,200	0,300	-0,900	0,100	-0,300	0,000	-0.100
INPUT POWER	1.000	0,955	0,933	1,230	0,977	1,072	1,000	1,023
INPUT PHASE IN DEGREES	5,00	275,00	265.00	175.00	275,00	185,00	175,00	85,00
OUTPUT POWER	8,084	0,020	0,047	0,027	0,004	0,002	0,001	0.005
OUTPUT IN DB RELATIVE TO ACTUAL TOTAL INPUT	-0,057	-26,039	-22,416	-24,745	-32,604	-36,069	-41,934	-32,236
OUTPUT IN DB RELATIVE TO TOTAL POWER WITH EQUAL INPUT POWERS	0,045	-25,937	-22,313	-24,643	-32,502	-35,967	-41,832	-32,133
ISOLATION IN DB RELATIVE TO PRIMARY OUTPUT PORT	-0,000	25,982	22,359	24,688	32,547	36,013	41,877	32,179

Appendix B
EFFECTS OF HYBRID UNBALANCE ON LOSS AND ISOLATION

The approach to determining the effects of hybrid unbalance on loss and isolation will be to start with a minimum-size combiner matrix (quadrature hybrids will be assumed) and, using results for smaller matrices, develop the output signals for the next larger size matrices. All hybrids will be assumed identical.

RESULTS WHEN $N = 2^k$ WITH $N = 2$ AND $k = 1$

Consider an output matrix consisting of a single quadrature hybrid (Fig. B1) of perfect match and infinite isolation with transmitted and coupled voltage coefficients T and C respectively. Assume conservation of energy, so that $T^2 + C^2 = 1$. Input signals are to be of uniform amplitude (chosen as unity magnitude for convenience) with phase at input ports 1 and 2 of θ_1 and θ_2. Since the individual inputs are of unity magnitude, the input power to a matrix will be N and thus will be 2. The output signals of this hybrid are

$$V_1 = Te^{j\theta_1} + jCe^{j\theta_2}$$

and

$$V_2 = jCe^{j\theta_1} + Te^{j\theta_2}.$$

If $\theta_1 = 0$ and $\theta_2 = 3\pi/2$, the principal output port is port 1, with $V_1 = T + C$ and $V_2 = j(C - T)$.

The worst case loss and isolation are

$$\text{worst-case loss (dB)} = -10 \log_{10} \frac{(T + C)^2}{2} = -10 \log_{10} \frac{(T + C)^{2k}}{N}$$

and

$$\text{worst-case isolation (dB)} = 10 \log_{10} \frac{(C + T)^2}{(C - T)^2}.$$

RESULTS WHEN $N = 2^k$ WITH $N = 4$ AND $k = 2$

Combine the output of two matrices as considered in the preceding section (each matrix being a single hybrid) into two hybrids as shown in Fig. B2.

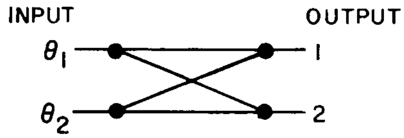


Fig. B1 — Single quadrature hybrid (two-by-two matrix)

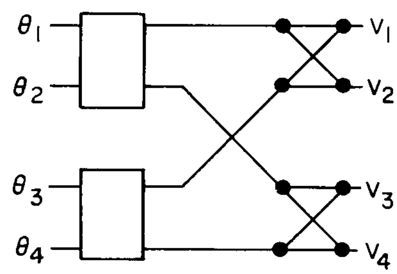


Fig. B2 — Matrix with the outputs of two single-hybrid matrices combined into two hybrids (four-by-four matrix)

Let $\theta_3 = \theta_1 + \phi$ and $\theta_4 = \theta_2 + \phi$; thus

$$V_1 = T(T + C) + jCe^{j\phi}(T + C),$$

$$V_2 = jC(T + C) + Te^{j\phi}(T + C),$$

$$V_3 = Tj(C - T) + jCe^{j\phi}j(C - T),$$

and

$$V_4 = jCj(C - T) + Te^{j\phi}j(C - T).$$

To maximize the signal at output port 1, let $\phi = 3\pi/2$ (that is, $\theta_1 = 0$, $\theta_2 = \theta_3 = 3\pi/2$ and $\theta_4 = \pi$); thus the output power at each port becomes

$$P_1 = |T^2 + TC + CT + C^2|^2 = (C + T)^4$$

$$P_2 = |j(TC + C^2 - T^2 - TC)|^2 = [(C - T)(C + T)]^2,$$

$$P_3 = |j(TC - T^2 + C^2 - CT)|^2 = [(C - T)(C + T)]^2,$$

and

$$P_4 = |-C^2 + TC + CT - T^2|^2 = (C - T)^4.$$

The maximum power will thus exit at port 1, with the next strongest signals at ports 2 and 3. Port 4 will have the weakest signal. Normalized to the output power at port 1, the relative output powers will be

$$\frac{(C + T)^4}{(C + T)^4} = 1 > \frac{(C + T)^2(C - T)^2}{(C + T)^4} \geqslant \frac{(C - T)^4}{(C + T)^4} .$$

Loss (relative to total input power) and isolation would then be

$$\text{worst-case loss (dB)} = -10 \log_{10} \frac{(C + T)^4}{4} = -10 \log_{10} \frac{(C + T)^{2k}}{N} ,$$

$$\text{worst-case isolation (dB)} = 10 \log_{10} \frac{(C + T)^2(C + T)^2}{(C - T)^2(C + T)^2} = 10 \log_{10} \frac{(C + T)^2}{(C - T)^2},$$

and

$$\text{best-case isolation (dB)} = 10 \log_{10} \frac{(C + T)^4}{(C - T)^4} = 10k \log_{10} \frac{(C + T)^2}{(C - T)^2} .$$

RESULTS WHEN $N = 2^k$ WITH $N = 8$ AND $k = 3$

Combine the outputs of two $N = 4$ matrices into four hybrids as shown in Fig. B3 and let the input phases of the second combiner be a constant $\phi = 3\pi/2$ radians greater than the input phase of the first combiner matrix ($\theta_5 = \theta_1 + 3\pi/2$, $\theta_6 = \theta_2 + 3\pi/2$, $\theta_7 = \theta_3 + 3\pi/2$, and $\theta_8 = \theta_4 + 3\pi/2$). Then

$$P_1 = |T(C + T)^2 + jCe^{j\phi}(C + T)^2|^2 = (C + T)^6,$$

$$P_2 = |jC(C + T)^2 + Te^{j\phi}(C + T)^2|^2 = (C + T)^4(C - T)^2,$$

$$P_3 = |jT(C^2 - T^2) + j^2Ce^{j\phi}(C^2 - T^2)|^2 = (C + T)^4(C - T)^2$$

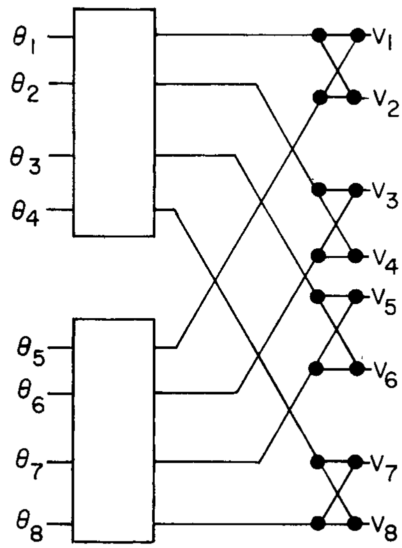


Fig. B3 — Matrix with the outputs of two $N = 4$ matrices combined into four hybrids (eight-by-eight matrix)

$$P_4 = |j^2C(C^2 - T^2) + jTe^{j\phi}(C^2 - T^2)|^2 = (C + T)^2(C - T)^4,$$

$$P_5 = |jT(C^2 - T^2) + j^2Ce^{j\phi}(C^2 - T^2)|^2 = (C + T)^4(C - T)^2,$$

$$P_6 = |j^2C(C^2 - T^2) + jTe^{j\phi}(C^2 - T^2)|^2 = (C + T)^2(C - T)^4,$$

$$P_7 = |jT(C - T)^2 + j^2Ce^{j\phi}(C - T)^2|^2 = (C + T)^2(C - T)^4,$$

and

$$P_8 = |j^2(C - T)^2 + jTe^{j\phi}(C - T)^2|^2 = (C - T)^6.$$

The maximum signal will exit at port 1, with ports 2, 3, and 5 having the next strongest signal and port 8 having the weakest signal. The relative output powers will be

$$1 > \frac{(C + T)^4(C - T)^2}{(C + T)^6} \geqslant \frac{(C + T)^2(C - T)^4}{(C + T)^6} \geqslant \frac{(C - T)^6}{(C + T)^6}.$$

Inspection of the outputs yields

$$\text{worst-case loss (dB)} = -10 \log_{10} \frac{(C + T)^6}{8} = -10 \log_{10} \frac{(C + T)^{2k}}{N},$$

$$\text{worst-case isolation (dB)} = 10 \log_{10} \frac{(T + C)^6}{(C + T)^4 (C - T)^2} = 10 \log_{10} \frac{(C + T)^2}{(C - T)^2} \quad ,$$

and

$$\text{best-case isolation (dB)} = 10 \log_{10} \frac{(C + T)^6}{(C - T)^6} = 10k \log_{10} \frac{(C + T)^2}{(C - T)^2} \quad .$$

SUMMARY

An approach to determining the effect of hybrid unbalance on loss and isolation has been demonstrated for matrices of quadrature hybrids having up to eight inputs and eight outputs ($N = 8$, $k = 3$). Other calculations not shown here verified that the same form is valid for $k = 4$ ($N = 16$). This approach, which can readily be extended to larger combiner matrices, indicates that the worst-case loss relative to the total input power is of the form

$$\text{worst-case insertion loss (dB)} = -10 \log_{10} \frac{(C + T)^{2k}}{N} = -20k \log_{10} \frac{(C + T)}{\sqrt{2}} \quad .$$

It also indicates that the worst-case isolation is independent of the size of the matrix (when $N = 2^k$) and is of the form

$\text{worst-case isolation (dB)} = 10 \log_{10} \left(\frac{C + T}{C - T} \right)^2$. When N is four or greater and only two output ports are needed, ports can be selected to obtain a best-case isolation even though the hybrids are unbalanced. This best-case isolation is of the form

$$\text{best-case isolation (dB)} - 10k \log_{10} \left(\frac{C + T}{C - T} \right)^2 \quad .$$

Appendix C
A SCHEME FOR INCREASING THE POWER-HANDLING CAPABILITY OF A BROADBAND POWER COMBINER/SWITCH MATRIX

Traveling-wave tubes have been developed that cover more than a rectangular-waveguide frequency band. In many practical transmission lines it appears that the broader the operating bandwidth of the transmission line, the lower the power-handling capability of the transmission line (for example double-ridge waveguide as compared with rectangular waveguide). A power combiner matrix using hybrids having different power and bandwidth capacities can be used to exploit the bandwidth capability of these tubes.

Figure C1 depicts a power combiner fed by an input-power-divider matrix, and eight parallel amplifiers. Switching is by selection of the input port; input 1 results in output at 8, input 2 results in output at 7, input 3 results in output at 6, etc.

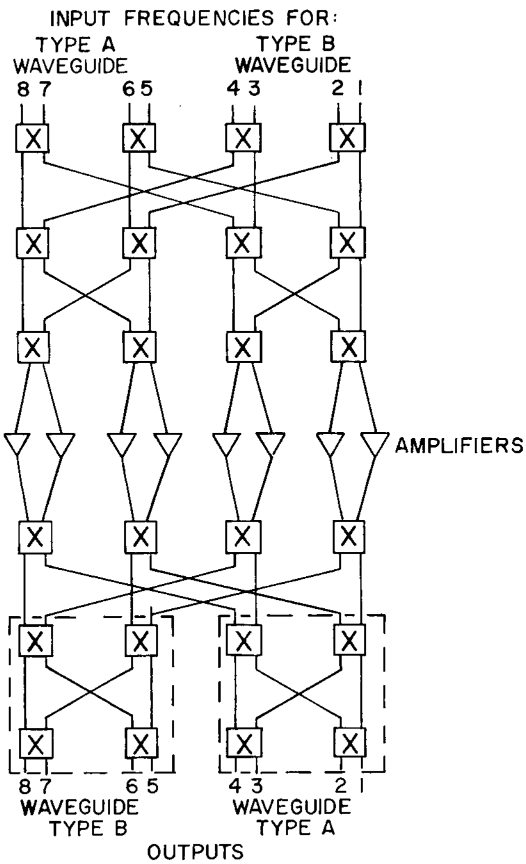


Fig. C1 — Broadband matrix with outputs in two waveguide bands

The combiner hybrids immediately following the amplifiers must cover the full desired bandwidth (that of the tubes), but they are required to handle only the power output of the two tubes feeding these hybrids; thus the power-handling capability can be 6 dB less than that of the final output hybrids. If higher-power narrower-bandwidth hybrids are employed as shown in Fig. C1, then the full frequency band of the amplifiers is available to power levels 6 dB higher than with only the broadband hybrids, provided these levels do not exceed the ratings of the output hybrids. However switching is now limited with regard to frequency: use of one group of inputs (such as inputs 1 through 4) results in signals from waveguide type-A outputs (5 through 8) while use of the second group of inputs (5 through 8) results in signals from waveguide type-B outputs (1 through 4). Specifications on the overall switch would in high probability be tightened, since the high-frequency waveguide would be below cutoff for the frequencies directed to the low-frequency waveguide outputs; leakage or scattered signals could thus be reflected directly back to the amplifiers.